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UNSTEADY HEAT TRANSFER IN WOODEN CYLINDRICAL SETS

A. Ya. Kuzin, N. A. Tsvetkov, A. N. Khutornoi, S. V. Khon, and T. A. Miroshnichenko

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A mathematical model and a numerical technique have been proposed for calculating the thermal state of a fragment of the outside log wall of a building. Temperature fields in homogeneous and inhomogeneous (filled with a warmth-keeping agent) wooden cylindrical sets at a variable heat load on the surface have been established and their comparative analysis has been performed.

Wood is now widely employed in residential and industrial building construction. This is due to its high sanitary and hygienic indices, accessibility to many regions of Russia, and high workability. According to [1], the use of homogeneous outside log walls in cold climatic zones requires an improvement of their thermotechnical characteristics. To improve thermal-protection properties of the outside wooden wall, a method has been proposed in [2] that which is based on filling the longitudinal axial opening of a homogeneous wooden log with a warmth-keeping agent. Since, with the aid of standard procedures from [1], it is impossible to establish the mechanism of unsteady heat transfer in inhomogeneous wooden structures, whose knowledge allows a purposeful improvement of their thermal-protection properties, this problem should be solved using mathematical modeling relying on adequate mathematical models and efficient methods of solution. In connection with this, the aim of the current study is the development of a mathematical model and a numerical technique for solving the problem of unsteady heat transfer in homogeneous and inhomogeneous (filled with a warmth-keeping agent) logs that are fragments of the outside log walls; the determination of the laws of unsteady heat transfer in them; and the comparative analysis of the efficiency of their thermal-protection properties.

Physicomathematical Statement of the Problem. Consideration is given to heat transfer in the radial section of a wood log with a warmth-keeping agent that is part of the outside wall enclosure, with a heat load on the boundary $r_\gamma(\varphi)$ varying over the circumferential coordinate φ (Fig. 1). The shape of a homogeneous log and warmth-keeping agent is represented by straight coaxial cylinders with radii R_1 and R_2. In the lower part of the log, there is a cut-out specified by the technological conditions of the assembly of the log wall. Known are the radii of the log and warmth-keeping agent, the distance between the centers O and O_1 of the sections of neighboring logs R_{OO_1}, and the thermophysical characteristics of wood and the warmth-keeping agent (the thermal conductivities in the radial (λ_r) and circumferential (λ_φ) directions, the density ρ, and the specific heat c). Conditions of the radiative-convective heat transfer are fulfilled on the external boundary of the radial section CD, conditions of convective heat transfer — on the internal boundary AB, and adiabatic conditions — on the lines AD and BC. Specified are the temperatures of external ($T_{g,e}$) and inside ($T_{g,ins}$) media, the heat-transfer coefficients on the outside (α_w) and inside (α_0) surfaces of a log, and radiation parameters of the outside surface of a log (ε_w) and of the external medium (ε_e).

The coordinate of the boundary of the upper log $r_\gamma(\varphi)$ in its lower part at the place of its junction with the lower log along the line AD is a variable dependent on the angle φ. The angle φ_s and the length of the radius vector OM of an arbitrary point M on the line AD with the angle $\varphi \in [-\varphi_s, \varphi]$ were determined from geometrical considerations on the basis of known R_1, R_{OO_1}, and φ using the equations

$$\varphi_s = \arcsin \sqrt{R_1^2 - (R_{OO_1}/2)^2}/R_1 \ , \quad r_\gamma(\varphi) = \sqrt{x_M^2 + y_M^2} \ ,$$

Tomsk State University of Architecture and Construction, 2, Solyanaya Sq., Tomsk, 634003, Russia; email: kaftgs@tsuab.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 79, No. 5, pp. 74–79, September–October, 2006. Original article submitted February 23, 2005.

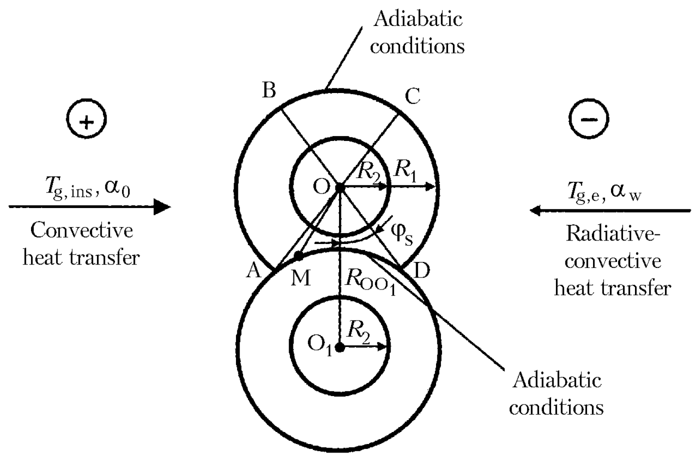


Fig. 1. Diagram of the radial section of a log with a warmth-keeping agent.

$$y_M = \frac{-R_{OO_1} + \sqrt{R_{OO_1}^2 - (1 + \tan^2 \varphi)(R_{OO_1}^2 - R_1^2)}}{1 + \tan^2 \varphi}, \quad x_M = -y_M \tan \varphi .$$

Heat transfer in the radial section of a log filled with a warmth-keeping agent was described using a mathematical model, written in the cylindrical system of coordinates, which consisted of two nonlinear two-dimensional, heat-conduction, equations for a homogeneous log and warmth-keeping agent:

$$(\rho c)_i \frac{\partial T_i}{\partial \tau} = \frac{1}{r} \frac{\partial}{\partial r} \left(\lambda_{r,i} r \frac{\partial T_i}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \varphi} \left(\lambda_{\varphi,i} \frac{\partial T_i}{\partial \varphi} \right), \quad i = 1, 2 , \tag{1}$$

with initial and boundary conditions

$$T_i \Big|_{\tau=0} = T_{in}(r, \varphi) , \quad i = 1, 2 ; \tag{2}$$

$$\lambda_{r,1} \frac{\partial T_1}{\partial r} \Bigg|_{r=r_\gamma} = \alpha_w (T_{g,e} - T_w) + \sigma \varepsilon_{eff} (T_{g,e}^4 - T_w^4) , \quad \varphi_s \le \varphi \le \pi - \varphi_s ; \tag{3}$$

$$\lambda_{r,1} \frac{\partial T_1}{\partial r} \Bigg|_{r=r_\gamma} = \alpha_0 (T_{g,ins} - T_0) , \quad \pi + \varphi_s \le \varphi \le 2\pi - \varphi_s ; \tag{4}$$

$$\frac{\partial T_1}{\partial r} \Bigg|_{r=r_\gamma} = 0 , \quad (2\pi - \varphi_s < \varphi < \varphi_s) \cup (\pi - \varphi_s < \varphi < \pi + \varphi_s) ; \tag{5}$$

$$T_i \Big|_{\varphi=0} = T_i \Big|_{\varphi=2\pi} , \quad i = 1, 2 ; \tag{6}$$

$$T_1 \Big|_{r=R_2} = T_2 \Big|_{r=R_2} ; \tag{7}$$

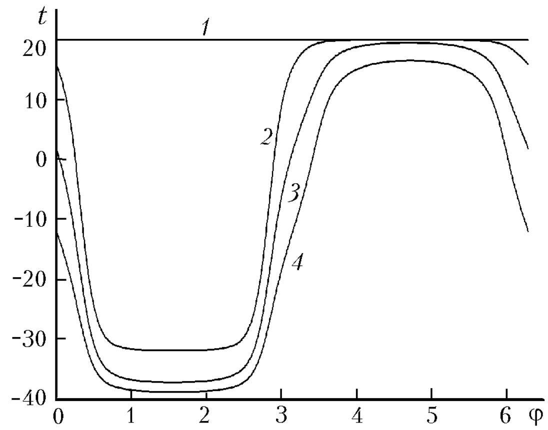


Fig. 2. Temperature distribution over the surface of a homogeneous log with respect to φ at various instants of time τ: 1) 0; 2) 1; 3) 5; 4) 72 h. t, $^{\circ}$C; φ, rad.

$$\lambda_{r,1} \frac{\partial T_1}{\partial r}\bigg|_{r=R_2} = \lambda_{r,2} \frac{\partial T_2}{\partial r}\bigg|_{r=R_2} . \tag{8}$$

Subscripts 1 and 2 in mathematical model (1)–(8) characterize a homogeneous log and a warmth-keeping agent. On the internal boundary of the latter, conditions (7) and (8) of the fourth kind are fulfilled. Boundary condition (6) is the periodicity condition. The function ε_{eff} is calculated from the Christiansen equation $\varepsilon_{\text{eff}} = (\varepsilon_{\text{e}}^{-1} + \varepsilon_{\text{w}}^{-1} - 1)^{-1}$.

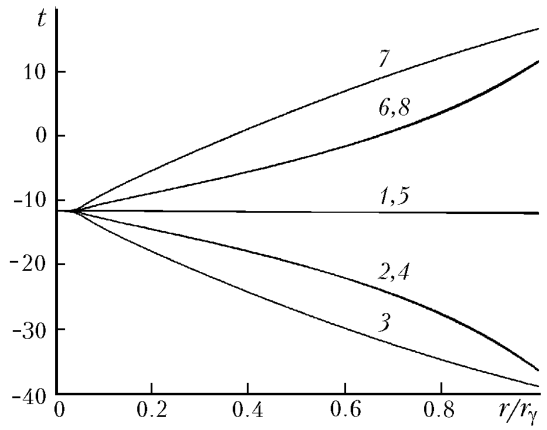
Method of Solution of the Problem and Results of Numerical Calculations. The problem was solved numerically using the splitting method of N. N. Yanenko [3]. One-dimensional equations of heat conduction in single-layer (in the direction φ) and two-layer (in the direction r) regions, obtained as a result of splitting, were calculated using the iteration-interpolation method [4, 5] with iterations with respect to coefficients with a specified accuracy. On the boundary $r = R_2$, use was made of special difference equations obtained by the iteration-interpolation method and with account for the difference in the thermophysical properties of wood and the warmth-keeping agent. Since the condition of symmetry at the center $r = 0$ is not fulfilled because of variability of the external heat load, the temperature in the direction r was calculated with the boundary condition of the first kind at $r = 0$. In order to determine the temperature at the center, a special iteration procedure was developed that is based on the solution, on each time layer, of the heat balance equation for an elementary cylinder with axis $r = 0$ and cross-sectional radius much smaller than the log radius.

The problem was solved numerically by the above algorithm using a program developed on the module principle in the FORTRAN programming language for a personal computer. Individual program modules were tested on the basis of analytical solutions known from the literature or obtained using the method of trial functions [5, 6]. The number of nodes of splitting the difference grid in the radial direction was $N_r = 51$ and in the circumferential direction $N_{\varphi} = 21$, and the time step was $h_{\tau} = 60$ sec. The time of calculating the datum variant up to $\tau_{\text{fin}} = 72$ h on a Pentium-4 personal computer was no longer than 3 min. To ease the analysis of calculated results it was assumed that the thermophysical characteristics of wood and the warmth-keeping agent are independent of the directions. For clarity, the initial and calculated temperatures will be represented in $^{\circ}$C.

The numerical study of the thermal state of a fragment of the outside log wall was carried out at the following parameters: $R_1 = 0.1$ m, $R_2 = 0.05$ m, $R_{\text{OO}_1} = 0.18$ m, $\lambda_1 = 0.14$ W/(m·K), $\rho_1 = 500$ kg/m^3, $c_1 = 2300$ J/(kg·K), $\lambda_2 = 0.04$ W/(m·K), $\rho_2 = 80$ kg/m^3, $c_2 = 1470$ J/(kg·K), $t_{\text{g.ins}} = 20^{\circ}$C, $t_{\text{g.e}} = -40^{\circ}$C, and $t_{\text{in}} = 20^{\circ}$C. The material of the homogeneous log was pine tree and that of the warmth-keeping agent was foamed polyurethane.

Figures 2–6 present results calculated for a homogeneous log, and Figs. 7 and 8 give, for comparison, results calculated for a log filled with a warmth-keeping agent.

The analysis of Fig. 2 indicates that temperature profiles on the surface are of a pronounced nonmonotonic character.The nearly horizontal segments of the temperature curves correspond to the boundaries of contact of the log



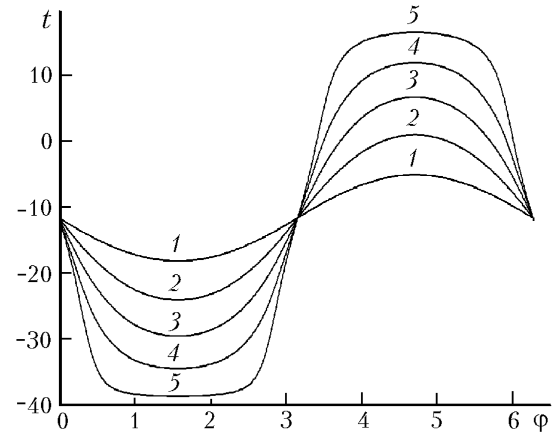


Fig. 3. Temperature distribution in a homogeneous log with respect to radii at τ_{fin} = 72 h for various φ: 1) 0; 2) φ_s; 3) π/2; 4) $(\pi - \varphi_s)$; 5) π; 6) $(\pi + \varphi_s)$; 7) 3/2 π; 8) $(2\pi - \varphi_s)$ rad; φ_s = 0.451 rad. t, ^{o}C.

Fig. 4. Temperature distribution in a homogeneous log with respect to φ at τ_{fin} = 72 h for various r: 1) 1/5 r_γ; 2) 2/5 r_γ; 3) 3/5 r_γ; 4) 4/5 r_γ; 5) r_γ. t, ^{o}C; φ, rad.

surface with external and internal air media. At τ_{fin} = 72 h, heat transfer practically reaches a steady state. Over this time, the temperature decreases from the initial t_{in} = 20^{o}C to 16.7^{o}C on the inside surface at φ = 3/2π and to –38.8^{o}C on the outside surface at φ = π/2. On adiabatic surfaces at φ = 0 and φ = π the temperature decreases to about –18.8^{o}C. The temperature variation is the most substantial for angles φ from the ranges $[-\varphi_s, \varphi_s]$ and $[\pi - \varphi_s, \pi + \varphi_s]$.

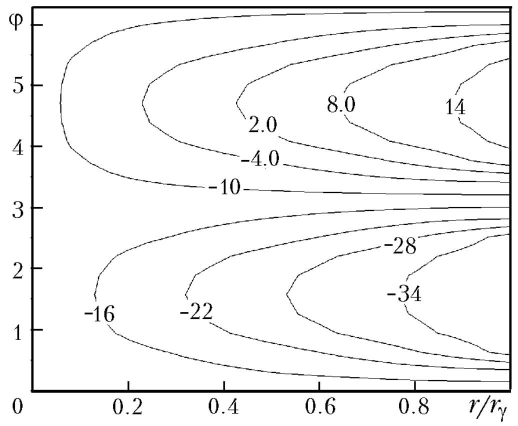
Figure 3 gives the temperature distribution with respect to radii of the most characteristic directions at τ_{fin} = 72 h (see Fig. 1). Clearly, the temperature distributions with respect to radii with angles 0 and π (curves 1 and 5), φ_s and $(\pi - \varphi_s)$ (curves 2 and 4), and $(\pi + \varphi_s)$ and $(2\pi - \varphi_s)$ (curves 6 and 8) are practically the same. The temperature difference along the length of radii with angles 0 and π (curves 1 and 5) is quite insignificant: it increases from 11.5^{o}C at r = 0 to 11.8^{o}C at $r = r_\gamma$. For other radii, the temperature difference is noticeable: 24.9^{o}C for curves 2 and 4, 23.2^{o}C for curves 6 and 8, 27.4^{o}C for curve 3, and 28.1^{o}C for curve 7. Dependences 1 and 5 are close to linear, and dependences 2–4 and 6–8 are close to parabolic, convex upward (curves 2, 4, and 7) and downward (curves 3, 6, and 8).

The temperature distributions with respect to φ for various r (see Fig. 4) show that, with a distance from the surface $r_\gamma(\varphi)$, the absolute values of the minima and maxima on the temperature curves corresponding to angles π/2 and 3/2 π decrease, and the temperature curves themselves become more smooth, which is due to the property of heat conduction lying in a strong smoothing and the time lag of characteristic features of the boundary functions with distance from the heat-transfer surface.

Isotherms in Fig. 5 characterize the temperature distribution over the cross section of a homogeneous log at τ_{fin} = 72 h. From Fig. 5 it follows that a major portion of the cross section is in the region of negative temperatures. The minimum and maximum values of the temperatures for various r correspond to angles π/2 and 3/2 π.

Figure 6 presents time dependences of the heat fluxes through the inside (the line AB) and outside (the line CD) open surfaces of a log. With a decrease in the temperature of the external air from 20^{o}C to –40^{o}C, the power of the heat flux through the outside surface of a log (curve 2) initially rises sharply, reaching at τ = 1 h its maximum value 44.7 W. Thereafter it begins to decrease, with the rate of decrease slowing down as a stationary value is approached. The power of the heat flux through the inside surface (curve 1) initially increases slowly, asymptotically approaching its stationary value. After heat conduction reaches a steady state, the powers of the heat fluxes through the inside and outside surfaces of a log become equal and amount to 9 W.

Figure 7 shows the temperature distributions with respect to radii of the most characteristic directions at τ_{fin} = 72 h for a log filled with a warmth-keeping agent. The designation of curves in Figs. 7 and 3 is the same. The



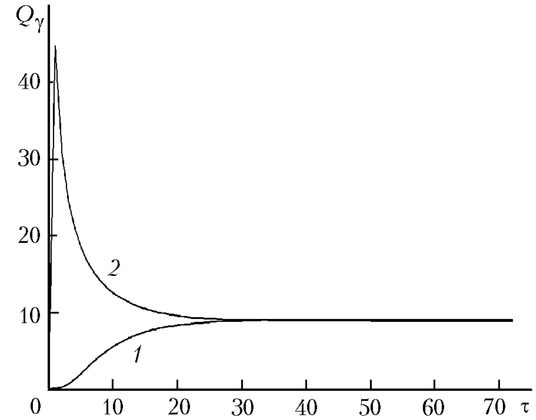
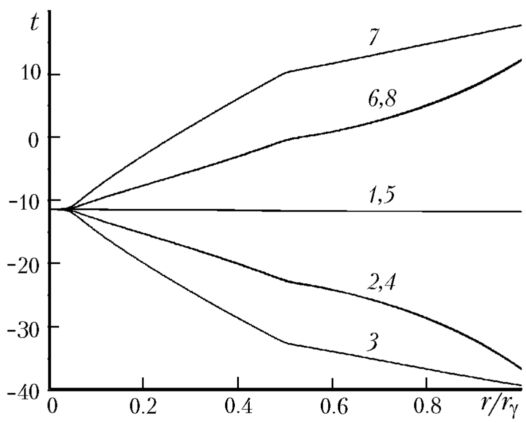


Fig. 5. Isotherms in the cross section of a homogeneous log. φ, rad.

Fig. 6. Heat fluxes through the inside (curve 1) and outside (curve 2) surfaces of a homogeneous log as functions of time. Q_γ, W; τ, h.



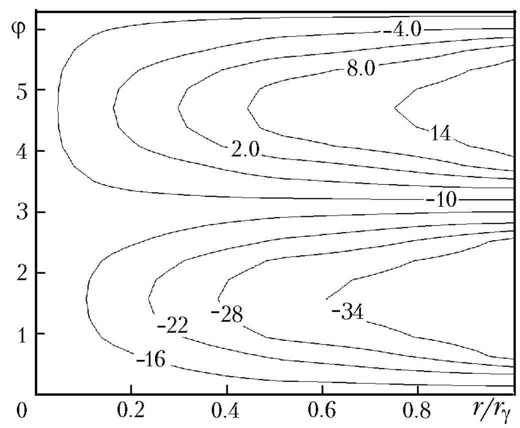


Fig. 7. Temperature distribution with respect to *r* in a log with a warmth-keeping agent at τ_{fin} = 72 h for various φ. Legend 1–8, same as in Fig. 3; φ_s = 0.451 rad. *t*, ^{o}C.

Fig. 8. Isotherms in the cross section of a log with a warmth-keeping agent at τ_{fin} = 72 h. *t*, ^{o}C; φ, rad.

analysis of Fig. 7 shows that curves of the temperature distribution with respect to radii on the boundary of contact of a homogeneous log filled with a warmth-keeping agent have an inflection. On this boundary, the temperature difference between a log filled with a warmth-keeping agent and a homogeneous log is at a maximum. The temperature difference is positive for the left half of the cross section facing the building and negative for the opposite half. Thus, for example, on the ray with the angle $\varphi = 3/2\ \pi$ it is 6.1^{o}C (the temperature values are 10.2^{o}C for the log filled with a warmth-keeping agent and 4.1^{o}C for the homogeneous log), and on the ray with the angle $\varphi = \pi/2$ it is –5.4^{o}C (−32.5^{o}C and −27.1^{o}C, respectively). The boundary of the sign reversal of the temperature difference for a log filled with a warmth-keeping agent agent and a homogeneous log lies practically at the center of the log. The temperatures at the center of the homogeneous log and the log filled with a warmth-keeping agent differ insignificantly and at τ_{fin} = 72 h are approximately equal to −11.5^{o}C.

Isotherms in the cross section of the log filled with a warmth-keeping agent in Fig. 8 support conclusions drawn from the analysis of Fig. 7. Comparison of Figs. 7 and 8 allows a rapid qualitative and quantitative assessment of the effect of a warmth-keeping agent on the temperature field in the cross section of a log. The presence of a warmth-keeping agent leads to a reduction in the heat flux through the open outside surface of a log from 9.0 to 7.2 W.

Thus, based on the mathematical modeling of unsteady heat transfer in homogeneous and inhomogeneous logs, the laws of the temperature distribution over cross sections have been disclosed, the heat fluxes through the inside and outside open surfaces have been determined, and the comparative analysis of their thermal-protection efficiency has been performed. The developed numerical technique provides rapid thermal diagnostics of the outside log walls, filled for warmth-keeping, with different thermophysical and geometric characteristics of wood and a warmth-keeping agent under different actual operational conditions.

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NOTATION

c, specific heat, J/(kg·K); h, step of the difference grid; N, number of nodes of the difference grid; Q, heat flux, W; r, radial variable of the cylindrical system of coordinates, m; R_1, log radius, m; R_2, radius of the cylindrical insert of a warmth-keeping agent, m; R_{OO_1}, distance between the centers of neighboring logs, m; r_γ, coordinate of the external boundary of a log dependent on φ, m; t, temperature, $^{\text{o}}$C; T, temperature, K; x_{M} and y_{M}, Cartesian coordinates of the point M, m; α, heat transfer coefficient, W/(m^2·K); ε, emissivity factor; ε_{eff}, effective function of radiation parameters; φ, circumferential variable of the cylindrical system of coordinates, rad; φ_{s}, angle of half the arc of adiabatic boundaries, rad; λ, thermal conductivity, W/(m·K); ρ, density, kg/m^3; σ, Stefan–Boltzmann constant, W/(m^2·K^4); τ, time, h. Subscripts: e, external medium; eff, effective; fin, final state; g, air; i, numbers of calculation regions; in, initial state; ins, inside medium; r, radial direction; s, adiabatic surface; w, outside surface of a log; φ, circumferential direction; γ, boundary; 0, inside surface of a log; 1, wood; 2, warmth-keeping agent.

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